

第八章

多元函数微分学及其应用

同步练习

8.1.一

$$1. \lim_{\substack{x \rightarrow a \\ y \rightarrow 0}} \frac{\arctan(xy)}{y} = \underline{\hspace{2cm}}$$

解 $\lim_{\substack{x \rightarrow a \\ y \rightarrow 0}} \frac{\arctan(xy)}{y} = \lim_{\substack{x \rightarrow a \\ y \rightarrow 0}} \frac{\arctan(xy)}{xy} \cdot x = 1 \cdot a = a$

$$2. \lim_{\substack{x \rightarrow \infty \\ y \rightarrow \infty}} \left(\frac{1}{x} \sin y + \frac{1}{y} \sin x \right) = \underline{\hspace{2cm}}$$

解 $\lim_{\substack{x \rightarrow \infty \\ y \rightarrow \infty}} \left(\frac{1}{x} \sin y + \frac{1}{y} \sin x \right) = 0 + 0 = 0$

$$\lim_{\substack{x \rightarrow \infty \\ y \rightarrow \infty}} \left(\frac{1}{x} \sin y + \frac{1}{y} \sin x \right) \neq \lim_{\substack{x \rightarrow \infty \\ y \rightarrow \infty}} \frac{1}{x} \lim_{\substack{x \rightarrow \infty \\ y \rightarrow \infty}} \sin y + \lim_{\substack{x \rightarrow \infty \\ y \rightarrow \infty}} \frac{1}{y} \lim_{\substack{x \rightarrow \infty \\ y \rightarrow \infty}} \sin x$$

8.1.—

$$3. \lim_{\substack{x \rightarrow 1 \\ y \rightarrow 0}} \frac{\cos(\pi x + y)}{\ln(x + e^y)} = \underline{\hspace{2cm}}$$

$$\text{解} \quad \lim_{\substack{x \rightarrow 1 \\ y \rightarrow 0}} \frac{\cos(\pi x + y)}{\ln(x + e^y)} = \frac{\cos(\pi + 0)}{\ln(1 + 1)} = -\frac{1}{\ln 2}$$

$$4. \text{函数 } z = \frac{y^2 + 2x}{y^2 - 2x} \text{ 的间断点为 } \underline{\hspace{2cm}}$$

$$\text{解} \quad y^2 = 2x$$

8.1.二

1. 求极限 $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{\sqrt{xy+4}-2}{xy}$

$$\begin{aligned}\text{解 } \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{\sqrt{xy+4}-2}{xy} &= \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{xy}{xy(\sqrt{xy+4}+2)} \\ &= \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{1}{\sqrt{xy+4}+2} = \frac{1}{4}\end{aligned}$$

2. 求极限 $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{x^2 y}{x^2 + y^2}$

解 令 $x = \rho \cos \theta$, $y = \rho \sin \theta$

$$\begin{aligned}\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{x^2 y}{x^2 + y^2} &= \lim_{\rho \rightarrow 0} \frac{\rho^2 \cos^2 \theta \cdot \rho \sin \theta}{\rho^2} \\ &= \lim_{\rho \rightarrow 0} \rho \cos^2 \theta \sin \theta = 0\end{aligned}$$

8.1.二

3. 求极限 $\lim_{\substack{x \rightarrow -\infty \\ y \rightarrow 0}} (1 + \frac{2}{x})^{\frac{x^2}{x+y}}$

解
$$\begin{aligned} \lim_{\substack{x \rightarrow -\infty \\ y \rightarrow 0}} (1 + \frac{2}{x})^{\frac{x^2}{x+y}} &= \lim_{\substack{x \rightarrow -\infty \\ y \rightarrow 0}} (1 + \frac{2}{x})^{\frac{x}{2} \cdot \frac{2x}{x+y}} = \lim_{\substack{x \rightarrow -\infty \\ y \rightarrow 0}} (1 + \frac{2}{x})^{\frac{x}{2} \cdot \lim_{\substack{x \rightarrow -\infty \\ y \rightarrow 0}} \frac{2x}{x+y}} \\ &= e^{\lim_{\substack{x \rightarrow -\infty \\ y \rightarrow 0}} \frac{2}{1 + \frac{y}{x}}} = e^2 \end{aligned}$$

4. 判断极限 $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{xy + y^2}{x^2 + y^2}$ 是否存在

解 令 $y = kx$,
$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{xy + y^2}{x^2 + y^2} = \lim_{\substack{x \rightarrow 0 \\ y = kx \rightarrow 0}} \frac{kx^2 + k^2x^2}{x^2 + k^2x^2} = \frac{k + k^2}{1 + k^2}$$

其值随 k 的不同而变化. 极限 $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{xy + y^2}{x^2 + y^2}$ 不存在

8.1.三

证明极限 $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{\sqrt{xy+1}-1}{x+y}$ 不存在

证 由于 $\frac{\sqrt{xy+1}-1}{x+y} = \frac{xy}{x+y} \cdot \frac{1}{\sqrt{xy+1}+1}$

$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{1}{\sqrt{xy+1}+1} = \frac{1}{2}$$

$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{xy}{x+y} = \lim_{\substack{x \rightarrow 0 \\ y=kx^2-x \rightarrow 0}} \frac{x(kx^2-x)}{x+kx^2-x} = \lim_{\substack{x \rightarrow 0 \\ y=kx^2-x \rightarrow 0}} \frac{kx^3-x^2}{kx^2} = -\frac{1}{k}$$

其值随 k 的不同而变化, 极限 $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{xy}{x+y}$ 不存在

因此极限 $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{\sqrt{xy+1}-1}{x+y}$ 不存在